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THERMAL EFFECTS DURING THE DEFORMATION OF GREY CAST IRON

ZDZISŁAW GABRYSZEWSKI and WIESŁAW ŚRÓDKA
Technical University of Wrocław, Institute of Materials Science and Applied Mechanics,
50-370 Wrocław, ul. Smoluchowskiego 25, Poland

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Abstract—Thermomechanical passive coupling in flake graphite cast iron has been studied. Results obtained in tension, compression, and shearing are given. Thermomechanical effect anisotropy in the elastic range has been observed during tension and compression.

1. INTRODUCTION

A large group of constructional materials are the bodies of which the mechanical properties depend on the loading sign—a symmetry of these properties does not occur (e.g. for the straight path of loading) in relation to the natural state. This group may include rocklike materials, grey cast iron, etc.

The constitutive equations that are commonly used do not determine, according to our experience, the behaviour of these materials. Therefore, these materials do not satisfy the fundamental assumptions, which are: the law of volumetric strains, and additivity of plastic and linear-elastic strains. The hitherto existing description of such media[1] does not seem satisfactory, both with regard to the accordance of theory with experiment, as well as to its physical principles. A great deal of information about constitutive equations can be obtained on the basis of the thermodynamics of the deformation process regarding the materials of this type. Due to this, the results of experimental tests for certain problems related to thermomechanical coupling have been presented in this paper.

The characteristic feature of the materials discussed and the main cause of the difficulty in their theoretical description is the occurrence of discontinuity in their structure—of internal cracking.

The flaky emission of graphite, occurring in the structure of cast iron, can be regarded, from the durability point of view, as internal cracks of the matrix, because the mechanical properties of graphite are very low. The microcracks of the matrix modify the macroscopic medium properties; during loading the material reveals, in certain directions, elastic nonlinearity and the ability to cause permanent changes of its volume. As mentioned above, these are the properties which considerably distinguish the internally cracked materials from the conventional elastic nonlinear materials. Despite the discontinuity and matrix cracking, it is possible to apply continuum mechanics to describe such a material. Vakulenko and Kachanov[2, 3], for example, introduce a vector field of displacement discontinuity into the cracks occurring during deformation, and then, with the help of this field, they form a certain tensor field describing the geometry of cracking. The introduced tensor is referred to as a crack density tensor, and is one of the local thermodynamic parameters of state. Such an application of the continuum mechanics of a continuous medium is related to the recently developed more general theory of materials with internal state variables[4–7].

Because cracked materials are characterized by the so-called anomalous isotropy, i.e. the dependence of some mechanical properties, e.g. yield stresses, on the loading sign and direction in the stress space, a question arises whether the thermal properties of such a medium, regarded phenomenologically, will also show anomalous isotropy when a continuous description is used. It appears that such tensor forms as $\beta_{ij} = -(\partial\sigma_{ij}/\partial T)_{\varepsilon_{kl}}$ or $\alpha_{ij} = (\partial\varepsilon_{ij}/\partial T)_{\sigma_{kl}}$ should also be dependent on the direction of loading—analogically to the mechanical properties.

However, there are no experimental data reported to confirm these suppositions. The expressions given above also determine the description of thermomechanical coupling. In the conventional elastic medium, the local relation between the temperature change Θ_e and the deformation carried out by the adiabatic method, ε_{ij}^e is described by the Kelvin formula

$$\Theta_e = \kappa_{ij}\varepsilon_{ij}^e \tag{1.1}$$

where

$$\kappa_{ij} = \frac{T_0}{\rho c_\varepsilon}\beta_{ij},$$

T_0 is the reference temperature, ρ the density, and c_ε the specific heat at permanent deformations, which, for normally isotropic bodies, is expressed by a linear relation between the temperature change Θ_e and the volume change ε_{kk}^e. It is evident that the dependence of the tensor β_{ij} on the path of loading also involves the dependence of temperature changes Θ_e on this path. This is an essential difference in relation to the conventional continuous medium and it concerns its thermal properties. It should also be taken into account that, in elastic continuous strains, there appears the other component of the temperature change Θ_d, associated with energy dissipation w_d. The total temperature change Θ is a sum of these two components

$$\Theta = \Theta_e + \Theta_d. \tag{1.2}$$

The latter part of the paper presents the test results concerning thermomechanical coupling occurring in flake graphite cast iron of tensile strength equal to 250 MPa, denoted as ZL250 according to Polish standards. The effect of recoverable strains and path of loading (straight) on the temperature change Θ_e, as well as the effect of permanent strains and loading directions on the temperature change Θ_d, are discussed.

2. EXPERIMENTAL TESTS

2.1. *Measuring technique*

Experiments were carried out on specimens made in the form of thin-walled tubes. The diameters and gauge length were $\phi = 41$ mm, $\phi = 38$ mm, and $l = 125$ mm, respectively. In order to measure the strains, 120 Ohm/6 mm wire gauges were used. The wire gauges were stuck on the outer surface of the specimen in the directions where principal deformations were anticipated. The temperature change induced by loading was measured with the use of the NTC210 type thermistor attached to the outer surface of the specimen in the middle of its length, as well as a specially constructed amplifier. However, such a measurement of the temperature of the real deformation process, treated as adiabatic, must satisfy certain conditions to ensure its correctness. The dynamics of the loading process are due to the occurrence of a certain difference between the temperature of the specimen protected adiabatically and the temperature indicated by the measuring instrument. The difference results from two basic reasons: the heat exchange between the specimen and the environment, as well as the measuring circuit inertia. In turn, the heat exchange is mainly caused by conductivity and convection. The influence of convection on temperature may be taken into consideration, owing to the fact that the temperature of the nonloaded specimen in which the possibility of heat exchange has been eliminated by conductivity (the specimen is positioned on a thermal insulator instead of being mounted in the machine holders) changes exponentially in time

$$\Theta_K = A \cdot e^{-Bt} \tag{2.1}$$

where A and B are constants.

For the specimens used here

$$B = \frac{1}{696}\,\mathrm{s}^{-1}. \tag{2.2}$$

It follows from (2.1) that during the basic tests, when the temperature of the specimen changes under the loading effect, the difference between the "adiabatic" temperature Θ_A (temperature of the adiabatically protected specimen) and the real temperature Θ_R (temperature changed by the convective heat exchange only) can be determined on the basis of the dependence

$$\mathrm{d}\Theta_K = -B\cdot\Theta_R\,\mathrm{d}t \tag{2.3}$$

where $\Theta_K = \Theta_A - \Theta_R$.

This difference, after time t^*, will equal

$$\Theta_K = -B\int_0^{t^*}\Theta_R\,\mathrm{d}t, \tag{2.4}$$

and it is easy to determine from the curves $\Theta_R(t)$ obtained during measurements. The relative temperature change caused by convection can be presented for an insignificant time interval Δt as

$$\frac{\Delta\Theta}{\Theta} \approx -B\cdot\Delta t.$$

Assuming the time Δt = 60 s, and considering eqn (2.2), the relative temperature change in the specimens applied here will be 8.6%.

The above estimation of the participation of convection in the heat exchange process is correct when the temperature change in the specimen is of a homogeneous nature. During the measurements, certain temperature gradients occur, and in this connection these data should be regarded approximately. The effect of heat conductance in the axial direction on the errors of the temperature measurement of the prism specimen was dealt with by Farren and Taylor[8]. The solution of a conductance equation for such a specimen of the initial temperature T (homogeneous), the ends of which are in contact with the environment of the temperature $T_0 \neq T$, indicates the existence of a certain time interval t^* in which the relative temperature change of the middle cross-section of the specimen induced exclusively by conductance is relatively small, see Fig. 1.

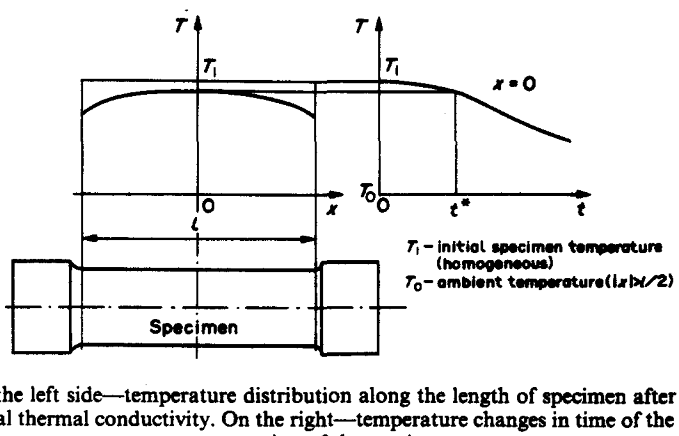

Fig. 1. On the left side—temperature distribution along the length of specimen after t^* associated with uniaxial thermal conductivity. On the right—temperature changes in time of the middle cross-section of the specimen.

For the temperature change by 0.6%, the time t^* in [8] was determined as

$$t^* = 0.014\,\rho c l^2/k$$

where ρ is the density, c the specific heat, l the length of sample measuring section, and k the thermal conductance coefficient. If the loading is completed in this time interval, the errors caused by heat conductance can be omitted and the deformation process occurring in the middle cross-section of the specimen can be practically regarded as adiabatic. The experimental results given in the above study indicate that the time t^*_{exper} is greater than the theoretically calculated time t^*. It follows from this that if measurements are carried out in the time interval t^*, the temperature measured (with the lack of convection) in the middle section of the specimen corresponds to the adiabatic process. In order to determine the effect of conductance on the correctness of the temperature change measurement, the following auxiliary tests were carried out. A certain constant force was quickly applied to the specimen (in about 3 s). The obtained dependence of temperature changes on the time for the middle section of the specimen indicates that after 60 s the temperature decreased with regard to the maximal rise Θ^* by about 10%, see Fig. 2. This increment is caused by convection and conductance. The earlier estimated temperature increments of about 8.6% induced by convection make it possible to state that temperature increments in the middle section of the specimen, caused by conductance are, after 60 s, smaller than 2%, and the effect of conductance can be omitted. The influence of convection can be taken into account by reading-out the temperature changes of the specimen according to the formula (2.4), whereas the largest measuring error of the "adiabatic" temperature is due to the inertia of the measuring circuit. The estimation of the errors due to the inertia of the measuring circuit was carried out by observing the state of temperature indications given on the curves in Fig. 2. It follows from these curves that the time which is needed to obtain the maximal indication Θ^* (corresponding to the moment when the real temperature is equal to that indicated) is about 15 s, but after about 3 s, the indications of the meter do not differ more than 10–15% from the value of Θ^*.

It is also worth noting that, due to the inertia of the measuring circuit, the value of the temperature being indicated Θ^* is lower than the maximal real temperature. Extrapolation of the curve given in Fig. 2 shows that the real temperature change after the

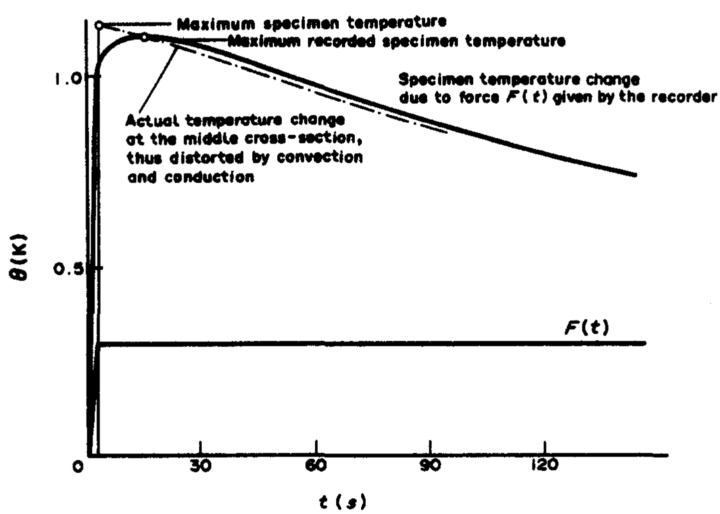

Fig. 2. Experimental curve of temperature changes of the middle cross-section of the specimen after being loaded with compressive force $F(t)$. The broken line shows schematically the real temperature changes. They are caused mainly by convection, and the effect of conductivity is relatively small here.

total loading of the specimen is higher by about 5% than the value Θ^*. Finally, taking into account that during the basic measurements the deformation rate did not exceed $3\times10^{-4}\ s^{-1}$ so the relative error temperature measurement after 60 s (resulting from the inertia of the instrument and heat exchange) was lower than 15%. In such a time interval the measurements were carried out.

The stresses, deformations, and temperature changes were measured on three straight paths of loadings—tension, compression, and torsion (simple shear). The tests carried out on each of these paths of loading were repeated on five specimens, which enables a statistical analysis of the obtained results. Of particular importance are comparisons of the coefficients obtained during the tension and compression tests, characterizing the mechanical and thermal properties of cast iron. To test the hypotheses concerning the equality of means $\bar{x}$ and $\bar{y}$, the following formula was applied

$$u=(\bar{x}-\bar{y})(n_1s_1^2+n_2s_2^2)^{-1/2}\left[\frac{n_1n_2}{n_1+n_2}(n_1+n_2-2)\right]^{1/2} \tag{2.5}$$

where

$$s_1=\left[\frac{1}{n_1}\sum_{i=1}^{n_1}(x_i-\bar{x})^2\right]^{1/2},\qquad s_2=\left[\frac{1}{n_2}\sum_{i=1}^{n_2}(y_i-\bar{y})^2\right]^{1/2}$$

of Student's t distribution with (n_1+n_2-2) degrees of freedom.

2.2. *Experimental results*

2.2.1. *Structural strain.* The typical tension, compression and, torsion curves are given in Figs. 3–5.

The characteristic feature of the obtained curves is that along the principal directions in which the deformations are negative, their dependence on stress is initially quasilinear

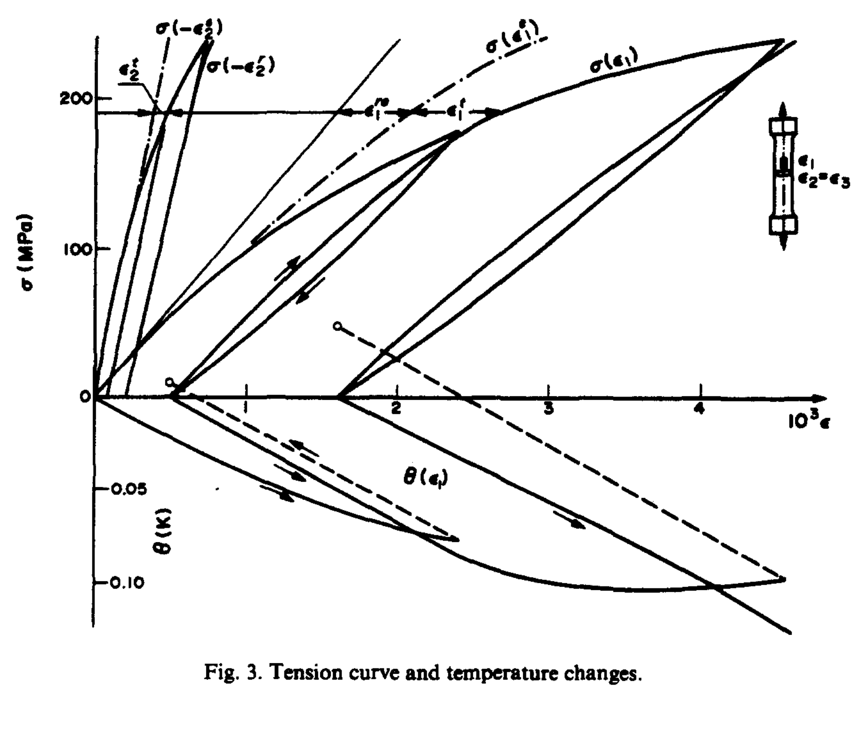

Fig. 3. Tension curve and temperature changes.

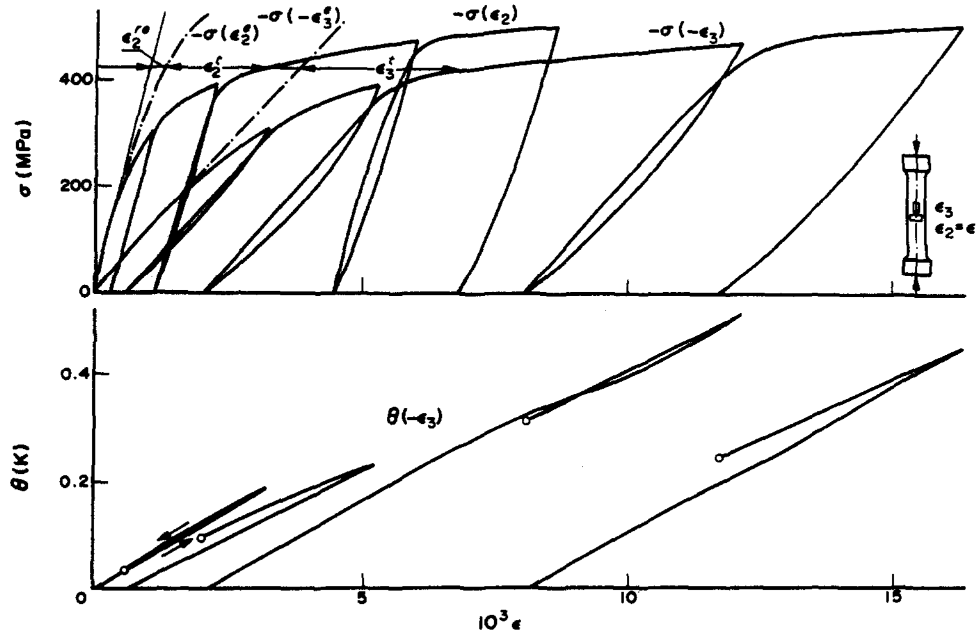

Fig. 4. Compression curve and temperature changes.

in the range of recoverable strains. The quasilinear dependences are the following: $\sigma_{\text{longit}}\,(\varepsilon_{\text{later}})$ in tension, $\sigma_{\text{longit}}\,(\varepsilon_{\text{longit}})$ in compression, and $\sigma_3(\varepsilon_3)$ in torsion. In the principal directions in which the strains are positive, the dependence of stress on these strains is nonlinear from the start [$\sigma_{\text{longit}}\,(\varepsilon_{\text{longit}})$ during tension, $\sigma_{\text{longit}}\,(\varepsilon_{\text{later}})$ in compression, and $\sigma_1(\varepsilon_1)$ in torsion]. This effect seems to be associated with the elastic "loosening" of the material. It is known and described especially with regard to rocklike materials, as well as to grey cast iron with large deformations.

The measurements carried out by us make it possible to state that this phenomenon occurs from the very start of loading, i.e. in the range of recoverable strains, which confirms the results of tension and compression tests presented by Gilbert[9].

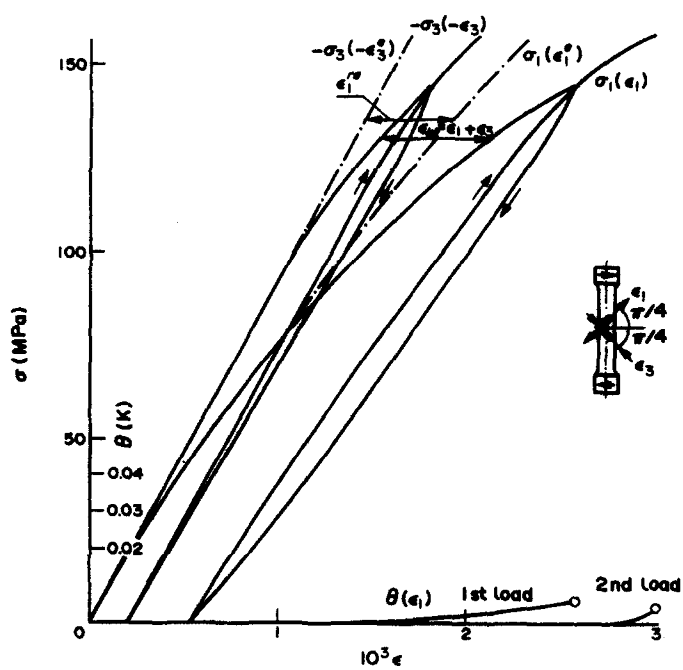

Fig. 5. Relations between principal stresses and corresponding principal strains during torsion. The temperature was drawn in the function ε_1.

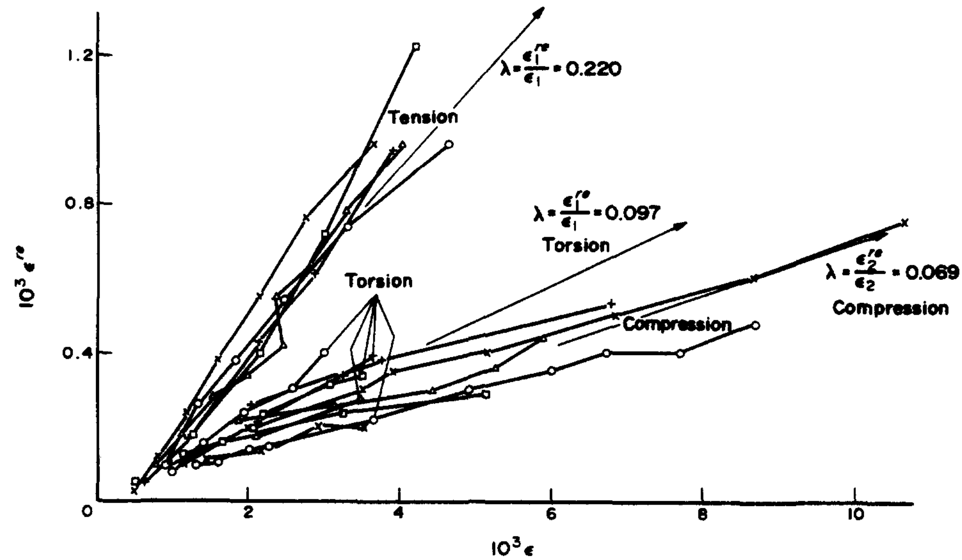

Fig. 6. Elastic structural strain ε_i^{re} during tension, compression, and torsion.

Let the principal elastic strain ε_i^e ($i = 1, 2, 3$) be called the deformation which occurs when an assumption is made that the medium is linearly elastic, whereas the principal elastic structural strain ε_i^{re} is the difference between the total principal recoverable strain in the direction "i" and the strain ε_i^e. This amount of strain is due to graphite-associated volume change and, in what follows, it will be briefly referred to as the "structural strain". Let us assume that $\varepsilon_i^{re} > 0$ when the total principal strain $\varepsilon_i > 0$, whereas in the ith principal direction in which $\varepsilon_i < 0$, the strain $\varepsilon_i^{re} = 0$.†

In order to determine the character of the relation between these deformations and their dependence on the path of loading, the measured strains ε_i^{re} as a function of the proper total principal strains ε_i are presented here.

The curves $\varepsilon_i^{re}(\varepsilon_i)$ for tension, compression, and torsion given in Fig. 6 are characteristic of two features: the possibility of the linear approximation of the functions as well as the dependence of the directional coefficient of these straight lined on the path of loading

$$\lambda = \frac{\varepsilon_i^{re}}{\varepsilon_i}, \quad \text{"}i\text{" should not be summed over.} \tag{2.6}$$

We will continue to refer to the coefficient λ as the loosening parameter. The values λ determined experimentally are 0.220 during tension (which means that the elastic structural strain is over one-fifth of the total strain), 0.097 during torsion, and 0.069 during compression. The direction in which the structural strain occurs during tension is the axial direction, during compression, perpendicular to the axis, and during torsion—the principal direction in which the strain is positive—inclined at the angle of $\Pi/4$ to the cross-section surface and tangent to the specimen surface. At advanced loading, besides the recoverable strains there occur permanent structural strains ε_i^{rr}, together with plastic strains.

The specific mechanism of deformation in grey cast iron accounts for the difficulties in the experimental measurement of volume changes. In tensile and compressibility tests the experimental determination of dilatation ε_{kk} is relatively simple. Lateral (measured) and radial (nonmeasured) strains are principally identical in tension and compression. During torsion, in addition to strains ε_1 and ε_3 (see Fig. 5.), it is also necessary to know the radial strain ε_2. Its measurement is extremely difficult to get on thin-walled specimens. It follows from the constitutive equations for cast iron ZL250 given in [1] that, during torsion, cast iron does not deform in the radial direction. On this basis, it has been assumed

† More precisely $\varepsilon_i^{re} \approx 0$ because precise tests indicate that a certain small nonlinearity of the dependence of stress-elastic strains in the principal direction in which $\varepsilon_i < 0$ suggests a "negative loosening". In the above-mentioned paper, Gilbert explains this by the closing of cracks occurring in graphite flakes.

Table 1. Values of parameter u (given by eqn (2.5)) and its comparison with the values of t Student variables for three significance levels: 0.05, 0.01, 0.001. With eight degrees of freedom, the values of the t Student distributional variable are $t^{0.05} = 2.306$, $t^{0.01} = 3.555$, and $t^{0.001} = 5.041$

	E_0	ν	$\frac{\Theta_0}{\varepsilon^e_{kk}}$	κ	Yield stress
u	1.468	8.466	8.432	2.478	7.59
$t^{0.05}$	$u < t^{0.05}$	$u > t^{0.05}$	$u > t^{0.05}$	$u > t^{0.05}$	$u > t^{0.05}$
$t^{0.01}$	$u < t^{0.01}$	$u > t^{0.01}$	$u > t^{0.01}$	$u < t^{0.01}$	$u > t^{0.01}$
$t^{0.001}$	$u < t^{0.001}$	$u > t^{0.001}$	$u > t^{0.001}$	$u < t^{0.001}$	$u > t^{0.001}$

that during torsion $\varepsilon_{kk} = \varepsilon_1 + \varepsilon_3$. Unfortunately, in the paper referred to, elastic structural strains are not taken into account, and that is why the omission of radial deformations is a certain simplification.

2.2.2. *Anisotropy of thermomechanical effect.* Temperature changes by strain during tension and compression in the elastic range are approximately proportional to longitudinal strains. During torsion within the elastic range of deformation, temperature changes remain equal to zero. The directional coefficients of the quasilinear section of these curves $\theta = \Theta_e/\varepsilon_{\text{longit}}$ differ considerably during tension and compression; with the same absolute value of the longitudinal strain, the temperature change during compression (with regard to the absolute value) is almost twice as high as the temperature change of the specimen tensioned.

Similarly, the temperature change to volume change ratio $\Theta_e/\varepsilon^e_{kk}$ during compression is about twice as high during tension.

The experimental data given above were obtained statistically using eqn (2.5). The calculation results are given in Table 2. The average quantities of the measured parameters obtained during tension and compression (Table 1) were compared for three significance

Table 2. Quantities characterizing the mechanical and thermal properties of cast iron, obtained experimentally. E_0 is the limit value of Young's modulus at loading equal to zero. During compression, this modulus is constant. The parameters θ, $\Theta_e/\varepsilon_{kk}$, and κ are given for loading equal to the yield point

	Specimen number	$10^{-5}\,E_0$ (MPa)	θ (K)	$\frac{\Theta_e}{\varepsilon^e_{kk}}$	κ (K)	Yield stress (MPa)
Tension	1	1.09	−32	−60.0	−79.0	112
	2	1.04	−26	−46.3	−71.3	120
	3	1.26	−29	−46.7	−73.6	117
	4	1.21	−33	−68.9	−88.2	90
	5	1.16	−42	−60.7	−82.2	70
	mean	1.152	−32.4	−56.5	−78.9	102
	95% confidence interval	0.11	7.5	12.2	8.5	26
Compression	1	1.07	−59	−125	−91.3	200
	2	1.12	−53	−136	−84.1	170
	3	1.04	−57	−109	−104.5	200
	4	1.09	−58	−114	−101.6	225
	5	1.11	−60	−147	−81.3	215
	mean	1.09	−57.4	−126	−92.6	202
	95% confidence interval	0.04	3.4	19.3	−12.8	26
Torsion	1			0	−93.1	90
	2			0	−87.7	95
	3			0	−97.7	45
	4			0	−98.7	50
	5			0	−87.1	45
	mean			0	−92.9	65
	95% confidence interval				6.7	31

levels, α. Hence it follows that slope coefficients θ differ considerably during tension and compression similarly to $\Theta_e/\varepsilon_{kk}$, as well as the yield stresses. On the other hand, the initial value of Young's modulus E_0 during tension does not differ significantly from the value obtained during compression; similarly the reduction coefficients $\nu = |\varepsilon_{later}/\varepsilon_{longit}|$ during tension and compression, as well as the slope of curves $\delta\sigma_1/\delta\varepsilon_1$ and $\delta\sigma_3/\delta\sigma_3$ during torsion, are not significantly different at the loading approaching zero. This points to the continuity of the mechanical properties of cast iron at $\sigma \rightarrow 0$.

The difference between the coefficients θ during tension and compression does not, then, result from the faster volume increase during tension as compared with the volume decrease during compression. Neither does it result from any dissipative processes during tension, since the unloading of the sample loaded almost to its yield point causes the temperature to return to the starting value.

Considering this fact, it should be stated that cast iron does not conform to Kelvin's formula in the form applied to normally isotropic media, i.e. temperature changes are not proportional to volume changes. This has also been confirmed by the results of torsion tests. The elastic volume increase (see Fig. 5) is not accompanied by a temperature change. The observed dependence of the increase in temperature caused by elastic strain on the path of loading will also be reflected in the complex states of stress. In view of the small range of permanent strains in cast iron as compared with plastic materials, the contribution of the Θ_e component in the total temperature change Θ (1.2) is relatively high for cast iron. Thus, this aspect of the anomalous cast iron isotropy (thermal), must introduce significant discrepancies between experiment and the theory being valid for normal isotropic media.

3. THERMOMECHANICAL COUPLING IN GREY CAST IRON

3.1. *Elastic range*

It appears that the two above-mentioned characteristic features of the experimental results obtained, i.e. the existence of the elastic structural strain as well as the anisotropy of the thermomechanical effect, are interrelated. This means that the functional relation between κ_{ij} [eqn (1.1)] and the elastic structural strain should occur. It should be emphasized here that the lack of experimental data concerning the relation $\kappa_{ij} = \kappa_{ij}\ (\varepsilon_{kl}^{es})$ confirms its acceptance in the simplest possible form. Those simplifications also result from limited experimental possibilities, i.e. from a small number of paths of loading which can be put into effect; moreover, the results of tests of internally cracked materials are characterized by large scatter, which also affects the degree of the complexity of empiric relations. In the above, we will confine our considerations to the principal directions of strains. Let us assume, according to what results from the curves in Figs. 3–5, that the principal values of the tensor κ_{ij} depend on the following:

(a) signs of the corresponding principal values of tensor ε_{ij},
(b) loosening parameter λ (2.6), and moreover
(c) absolute temperature T.

Let us write it in the following form

$$\kappa_i = H(\varepsilon_i) \cdot \kappa(\lambda, T) + H(-\varepsilon_i) \cdot \kappa(\lambda = 0, T), \tag{3.1}$$

where H is Heaviside's function. The influence of the (straight) path of loading was taken into account by introducing the parameter λ.

As a parameter determining the path of loading in the plane state of stress, the expression $\sigma_0/|\sigma_m|$ is commonly used, where σ_0 is the mean stress, and $|\sigma_m|$ is the largest principal stress, due to its absolute value. The curve $\lambda(\sigma_0/|\sigma_m|)$, plotted on the basis of three points (tension, compression, shearing) similar to the analogical curve plotted on the basis of a considerably larger number of points for elastic structural strains (given in [1]), indicates the univocal character of this function. This is based on assumption (b). Let us add that, besides the above mentioned, there are other possibilities of determining the path of loading, e.g. with the help of the Loge–Nadaj parameter.

The function $\kappa(\lambda, T)$ occurring in eqn (3.1) may be determined at $T = T_0$ on the basis of the experimental results obtained. By substituting (3.1) into (1.1), we obtain the temperature changes Θ_e during tension, compression, and shearing, respectively, at the ambient temperature T_0:

$$\Theta_e = \kappa(\lambda = 0.020)\cdot\varepsilon_1^e + 2\kappa(\lambda = 0)\cdot\varepsilon_2^e$$

$$\Theta_e = \kappa(\lambda = 0)\cdot\varepsilon_3^e + 2\kappa(\lambda = 0.069)\cdot\varepsilon_1^e$$

$$\Theta_e = \kappa(\lambda = 0.097)\cdot\varepsilon_1^e + \kappa(\lambda = 0)\cdot\varepsilon_3^e.$$

It has been assumed that $\varepsilon_1^e \geqslant \varepsilon_2^e \geqslant \varepsilon_3^e$.

From these equations, it is possible to calculate $\kappa(\lambda = 0.220)$, $\kappa(\lambda = 0.097)$, and $\kappa(\lambda = 0.069)$, providing that the value $\kappa(\lambda = 0)$ is known. $\kappa(\lambda = 0)$ was determined indirectly by making use of the above stated continuity of the mechanical properties of cast iron at $\sigma_{ij} \to 0$. Because the values κ_i do not depend on the magnitude of the strain ε_i, but merely on its sign (and of course on the parameter λ at $T =$ const), therefore if in the direction "i" there occurs $\varepsilon_i < 0$, then on the basis of the assumptions concerning the mechanism of deformation as well as the independence of the material constants on the path of loading at $\sigma_{ij} \to 0$, the value $\kappa(\lambda = 0)$ can be determined by using the formula for a normally isotropic body

$$\kappa(\lambda = 0) = -\frac{T_0\cdot E\cdot\alpha}{(1-2\nu_0)\rho c_e}, \tag{3.2}$$

where α is the linear thermal expansion coefficient, E the Young's modulus during compression (equal to the limit value of Young's modulus during tension $E_0 = \lim_{\sigma\to 0} E$), and ν_0 the limit value of the coefficient of cross section area reduction of the sample at $\sigma \to 0$. In order that the obtained value $\kappa(\lambda = 0)$ may be comparable with the remaining values $\kappa(\lambda > 0)$ (on account of any possible systematic errors of the measuring method) this value was "measured" in the following way

$$\kappa(\lambda = 0) = \left(\frac{\kappa_{c-i}}{\kappa_{st}}\right)\cdot\kappa_{st}.$$

The coefficient $(\kappa_{c-i}/\kappa_{st})$ was calculated on the basis of (3.2) by inserting the proper constants for cast iron and steel, while κ_{st} was measured experimentally on 5 tension-subject and 5 compression-subject steel samples identical to those made of cast iron. In this way, a coefficient κ for four values of the λ parameter was obtained. The results are given in Table 2 and in Fig. 7. The curves in Fig. 7 suggest that the dependence $\kappa(\lambda)$ (at $T =$ const) can be assumed to be linear.

The paths of loading investigated here are characterized by the fact that, in addition to $\kappa(\lambda = 0)$, the knowledge of only one function $\kappa(\lambda > 0)$ is required (naturally on the straight path of loading $\lambda =$ const). Generally, there may be more directions in which a loosening occurs. Therefore, a question arises as to whether it is possible to apply the same function $\kappa(\lambda)$ to the subsequent directions in which a loosening occurs. Such a problem results from the fact that the magnitude of structural strains on which κ depends in the complex states (e.g. shearing) is not the superposition of structural strains for proper simple states (tension and compression), if such were the case, the structural strains during torsion would, then, be greater than during tension. Explanation of this problem requires experimental verification.

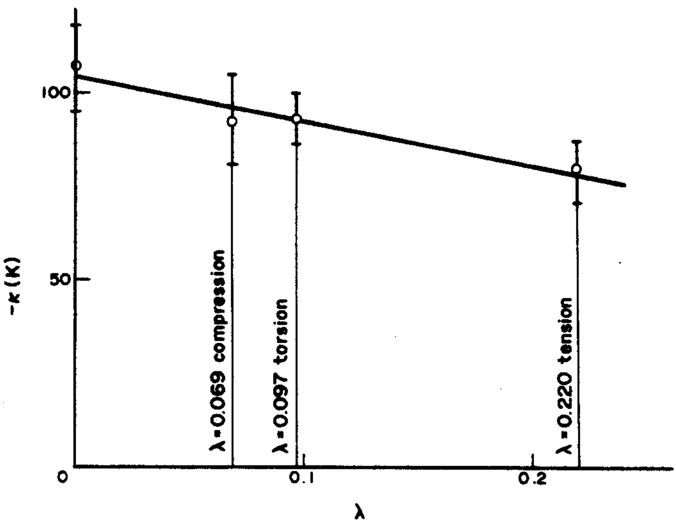

Fig. 7. Mean values of κ and 95% confidence intervals for four values of the loosening parameter λ occurring during tension, compression, and torsion.

3.2. *Energy dissipation in cast iron*

The ratio of the latent energy $w_H = w_t - w_d$ to the work on permanent strain $w_t = \int \sigma_{ij}\, d\varepsilon_{ij}^t$ (ε_{ij}^t—permanent strain, $w_d = \rho \cdot c \cdot \Theta_d$) shown on the curves of Fig. 4 does not deny the possibility of the assumption that w_H/w_t = const. The mean values of w_H/w_t for tension and compression do not differ significantly. The mean value for torsion differs significantly from the remaining values.

Significance tests were based on eqn (2.5). The mean values and 95% confidence intervals are marked in Fig. 8. The obtained results indicate that, in cast iron, the w_H/w_t is dependent on the path of loading. This is worth emphasizing since it has been experimentally proved that, for a number of elastic–plastic materials, the above ratio is independent of the direction of loading, e.g. for copper[10, 11] and for steel[11, 12].

4. DETERMINATION OF YIELD STRESSES

In mechanics there is a problem of experimental determination of the plasticity surface. It consists of finding such values of stress σ_{ij}^0 at which permanent strains reach a certain small, conventional value. Since the number of strain independent components is 6, there is a need for a scalar description of a permanent strain increment. The practical determination of the moment when the material changes into the plastic state requires a number of assumptions and simplifications to be made in the complex states of stress. The problem gets even more complicated in the case of anisotropic bodies. The simplifications used in such cases, as well as the critical notes concerning such simplifications, were given in [13].

The problems connected with the experimental determination of the yield surface make it necessary to search for simpler (burdened with a smaller number of assumptions) methods of their determination. For this purpose, the phenomenon of coupling between the strain field and the temperature field can be successfully applied[14]. The determination of the moment of crossing the yield surface in the quasidiabatic deformation process would be based on the assumption that this is accompanied by an additional temperature rise associated with energy dissipation. The measure of the yield point would then be such a value of stress σ_{ij}^0, which would correspond to the conventional value of this rise, i.e. the quantity of the dissipated energy w_d.

In the obtained results, it is an interesting fact that, despite the occurrence of loosening and the resulting nonlinearity of the material, there exists such a value of stress $\sigma_{ij}^0 \neq 0$ at which permanent strains ε_{ij}^t and an appreciable additional temperature increase Θ_d occur simultaneously. This not only means that the described method allows the yield surface to

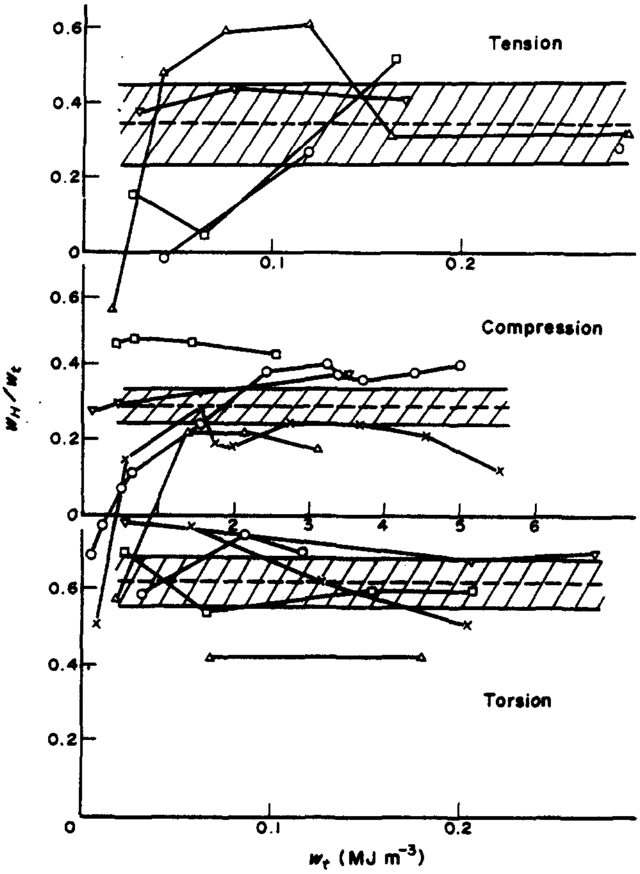

Fig. 8. Dependence of the w_H/w_t on w_t (w_H—latent energy, w_t—work on permanent strain) during tension, compression, and torsion. The broken line presents an approximation of the results by constant function. Shaded areas denote 95% confidence intervals.

be determined in a simple way, but also that it is possible to introduce the notion of a yield surface for grey cast iron.

5. CONCLUDING REMARKS

The presented results of the study into the thermomechanical coupling in grey cast iron, as well as its elastic properties, can be of some importance in the physical interpretation of the damage mechanism of this kind of material. The complex system of local highly heterogeneous strains in the metallic matrix space with adhering graphite inclusions, as well as the energy flow connected with the existence of cracks, greatly affect the mean temperature of the whole medium. These processes, initially reversible, show their distinct dependence on the ratios of principal stresses, i.e. the path of loading. It appears that the observed anisotropy of the thermomechanical effect in the initial, quasi-elastic, range of strains is a macroscopic reflection of a peculiar distribution of local strains in the matrix which determine the origin and development of cracks and, in consequence, the damage of the medium.

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